Using an Admittance Algorithm for Bone Drilling Procedures

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Abstract

Bone drilling is a common procedure in many types of surgeries, including orthopedic, neurological and otologic surgeries. Several technologies and control algorithms have been developed to help the surgeon automatically stop the drill before it goes through the boundary of the tissue being drilled. However, most of them rely on thrust force and cutting torque to detect bone layer transitions which has many drawbacks that affect the reliability of the process. This paper describes in detail a bone-drilling algorithm based only on the position control of the drill bit that overcomes such problems and presents additional advantages. The implication of each component of the algorithm in the drilling procedure is analyzed and the efficacy of the algorithm is experimentally validated with two types of bones.

Keywords: Assisted Surgery, Bone Drilling, Layer Detection.

1. Introduction

Drilling is a common procedure in bone surgery. It is used in orthopedic surgeries for fractures and bone reconstruction [1]; in neurological surgeries, for example in craniotomy and stereotactic surgery [2]; in ear surgeries, such as a cochleostomy for cochlear implantation [3] and in any otologic surgery [4]. In orthopedic surgeries, drilling is required in about 95% of post-trauma treatments and interventions, where holes are required to mount the screws needed to fix and correct bone fractures or to attach plates or other prosthetics.

Surgeons work very carefully, trying to control the penetration of the drill and avoid harming vessels and tissues located in the proximities of the bone. One of the main risks consists of projecting the drill bit beyond the structure being drilled. The safety of the patients relies on the experience of the surgeon, but it has been shown in experimental studies that surgeons drill beyond the far cortex by an average of 6.33 mm, depending on their age and experience [5].

Several technologies, each with different control algorithms and strategies, have been developed to help the surgeon to automatically stop the drill before it goes through the boundaries of the tissue being drilled. Most of them, implement algorithms and sensors to control the force of penetration and the torque of the drill.

In previous work we presented DRIBON [6, 7], a new automated-mechatronic drilling system which implemented an algorithm based on the position control of the drill bit. This strategy is completely different from the rest of the technologies found in the literature. The aim of this work is to describe in detail the differences between our approach and related work (Sections II and III), analyze in detail the implemented methodology (Section IV), and show its effectiveness when drilling different types of bones (Section V). The distinguishing feature of the approach is that it allows drilling in different bone types without changing the control algorithm or building a neural network based on experimental data. Additionally, layer transitions can be properly detected before bone breakthrough.

2. Related Work

This section presents a review of bone drilling methods and systems. They are grouped into categories depending on the strategy used to detect bone layer transitions.

2.1. Methodologies based on strong signal variation

One of the first methods in this group was presented by Brett et al. in 1995 [8]. They observed that the persistence presented in the increase of the torque and the decrease of the force at the moment of protrusion can be used to determine the exact moment at which to stop the drill. Their method determined that a change of layer occurs when threshold values in the increase of the cutting torque and decrease of the penetration force were reached.

In 1996, Allotta et al. [9] presented a new study focused only on a threshold value of the force signal. Results showed that the persistence presented in the increase of the torque and the decrease of the force at the moment of protrusion can be used to determine the exact moment at which to stop the drill. Their method determined that a change of layer occurs when threshold values in the increase of the cutting torque and decrease of the penetration force were reached.

In 1996, Allotta et al. [9] presented a new study focused only on a threshold value of the force signal. Results showed that the force profiles were usually less noisy than torque profiles, and so they decided to use force data exclusively for real-time detection. They define threshold values for the first derivative of the force of penetration, which in long bones are adjusted during the drilling of the first cortical wall in order to detect the transition of tissue and the breakthrough on the second cortical wall.
Ong and Bouazza-Marouf’s method [10], presented in 1998, implemented a modified Kalman filter to convert the profiles of differences in drilling force between successive samples and/or the drill bit rotational speed into easily recognizable and more consistent profiles. After discarding the rotational speed as a parameter for detecting the breakthrough due to its inconsistency at low speeds, they concluded that breakthrough occurs when the K-FDSS (Kalman processed Force Difference between Successive Samples) drops to zero.

In 2001, Hsu et al. [11] presented a modular mechatronic system for automatic bone drilling in surgery. One of its major features is that it is easy to “add-on” devices that are compatible with commercially available motor-driven drills. The electric current consumed by the drill’s DC motor is used as a sensing signal, and it was found that it has a direct relation with the cutting torque. A control box converts it into a voltage signal, and breakthrough is detected when the plot voltage vs. time presents a second peak and a sharp drop.

Lee et al. [12, 13] developed in 2003 an algorithm for force control and breakthrough detection, which was successfully implemented in a three-axis robotic bone drilling system [14]. It consists of an inner loop fuzzy controller for robot position control and an outer loop PD controller for feed unit force control. Actual thrust force is measured with a load cell. This signal is combined with a force equivalent to the drilling motor’s torque output and fed back into the outer loop. They based their breakthrough detection algorithm as a function of thrust force threshold information and trends in drill torque and feed rate.

In 2008 Coulson et al. [15] presented an application of an autonomous surgical robot system that was able to carry out the critical process of penetrating the bone tissue of the wall of the cochlea without penetrating the endosteal membrane located immediately inside the cochlea. A computer analyzes the force and torque imparted onto the drill in real time. Force is measured with a cantilever sensor on the drill bit and the torque is measured with the electrical power needed to turn the drill bit at a specified speed. Their control strategy stopped the burr when a force drop of 10 units was coupled with a torque rise of 10 units.

Other techniques considered the ultrasonically-assisted drilling of cortical bone. Alam et al. in 2010 [16] undertook an experimental investigation of forces and torque in conventional and ultrasonically-assisted drilling of cortical bone. Experiments were carried out with a bovine femur, from which cortical bones were cut. The results revealed that the penetration force and the torque dropped significantly when ultrasonic vibration was superimposed along the drill’s longitudinal axis. In addition to the fact that the force was halved for the range of the drilling speeds selected, it was observed that chip removal from the drilling site was improved.

In 2014, Sun et al. [17] presented a technique that includes a state recognition of bone drilling with audio signals. They analyzed the sound generated during operation via the Fast Fourier Transform, which presents different characteristics due to the different kind of bone along the path of the drilling process. The Exponential Mean Amplitude and the Hurst Exponent were used to validate the energy characteristics and stability of the audio signals. Based on this and a time counter, they developed an algorithm to recognize the drilling state, which runs in an Embedded Drilling State Monitor that realizes real-time performance.

In recent works, Hu et al. [18, 19] proposed a novel algorithm for state recognition based on the real-time force sensing of the drilling process. The algorithm takes into account the average value and the difference of the thrust force. It was implemented in a Robotic Spinal Surgical System (RSSS) to perform high-precision spinal surgeries using spherical and twist drills. To recognize the state transitions, threshold values were determined based on experiments with cattle’s vertebrae that are considered to have similar mechanical properties to human vertebrae.

2.2. Methodologies based on wavelet signal analysis

In 1998 Allotta and Colla [20] applied a wavelet-based controller to a mechatronic drill for orthopedic surgery. The penetration velocity of the drill is generated on the basis of the wavelet analysis of the thrust force signal and the controller is capable to fulfill three different tasks corresponding to different specifications of the hole to be done in a long bone, which are to stop the drill: 1st as soon as the first cortical wall is cut, 2nd just before the second cortical wall and finally, as soon as the second cortical wall is cut.

2.3. Methodologies based on fuzzy logic and neural network

Allotta et al. [21] presented such technology in 1997. They developed a hand-held drilling tool for orthopedic surgery that includes sensing, fuzzy reasoning and control capabilities in order to obtain controlled penetration. Fuzzy rules were different for each of the three different phases in which the drill can be found: end of first cortical wall, just before second cortical wall and the end of the second cortical wall.

In 2000, Kaburlasos and Petridis [22] successfully applied learning, classification and feature extraction techniques to the stapedotomy surgical procedure. They used force and torque data during drilling to estimate the thickness of the stapes bone by learning a linear mapping of force features to torque features. This learning was achieved by employing the two level fuzzy lattice (2L-FL) scheme for supervised clustering.

2.4. Methodologies based on medical image

Other approaches, though very different to previous ones, are based on the use of medical images to detect bone drilling states. In 2012, Luan et al. [23] presented a 3D navigation method providing a monitoring system for milling operations in spinal surgery using preoperative CT images and a registration method. During the operation, the cutting depth and especially the distance to the spinal canal are monitored. Warning messages are provided when the lamina is nearly milled through, so that the surgeons could take precautions to avoid surgical failures.

A bone-drilling state recognition algorithm based on image-force fusion was proposed in [24] for improved performance in
pedicle screw insertion surgeries. The short-time average magnitude of thrust force, the average energy of thrust force and their gradients are used to recognize drilling states. Besides, the preoperatively scanned medical images are combined with the real-time position of the operation tool. Fusing both information sources results in more accurate drilling procedures.

3. Bone Layer Detection Algorithms

Most of the drilling systems described in the previous section share a common characteristic: they impose a feed rate on the tool and observe changes in bone resistance to identify layer transitions. This is usually performed by measuring the thrust force (alone or in combination with other signals). Using a causality terminology common in control systems [25], we could describe this methodology as an impedance approach, in which velocity is the input imposed on the system and force is the output observed from the system.

To understand some practical limitations of this approach, Fig. 1 shows the resistance force measured during the drilling process of a bovine femoral shaft when a constant feed rate of 1 mm/s was set, and the drill bit was rotating at 3000 rpm. Fig. 1 also includes the result of applying zero-phase digital filtering, which does not introduce additional time delay, to qualitatively illustrate the signal-to-noise ratio.

Drilling the cortical bone requires a relatively high thrust force, and the two layer transitions (from the first cortical wall to the cancellous tissue, and from the second cortical wall to outside the bone) could be detected as an abrupt decrease in the force signal. However, this signal is very noisy and it must be conditioned before making any decision based on it. As a result, a certain delay is introduced after filtering the signal, which decreases the effectiveness in detecting layer transitions just before protrusion. This effectiveness also depends on the threshold value used to detect the abrupt decrease in force. High thresholds may lead to false transition detections, while conservative thresholds may result in detections that come too late.

To cope with these problems several authors have proposed some variants of this approach, combining the force with other records (e.g., cutting torque, motor current consumption or noise emitted) and building neural networks to adapt the algorithms based on experience.

In contrast to these algorithms, we propose using an admittance approach in which the force is the input imposed to the system and the velocity (the drill feed rate) is the output observed from the system. Such an approach was successfully developed by Louredo et al. [6, 7] and it is very similar to the manual way of performing the drilling process.

When people hold the tool in the hand and drill a hole into a wall, they do not know the actual penetration rate. They can only control—and with limitations—the thrust force. When the end of the wall is reached the breakthrough is felt by the user as an acceleration of the tool. In this situation, it is difficult to stop the drill because human beings do not react as quickly as automated robotic systems. An automated drilling tool, however, would perform the operation more precisely. To create such a tool and ensure its accuracy, our approach is to come up with an algorithm that understand the natural tendency of the tool, namely that the feed rate rapidly increases when it arrives at the end of the wall.

This approach has some significant consequences for the design of the tool. The most significant one is that the system has to be backdrivable. In other words, the tool must be able to move not only due to the forces commanded by a motor, but also due to external forces. For this reason, the admittance algorithm cannot be implemented with a device using, for example, worm drives.

The use of this natural tendency approach leads to some intrinsic benefits. For example, it is not necessary to use force sensors; only position sensors are needed. This results in more inexpensive designs and it avoids the problems associated with filtering the force signal (and delaying any consequent decision). Some other benefits, such as easy tuning even without experimental data, use of lower level of forces, etc., will become apparent once the behavior of these devices is explained in depth (see Section IV).

To illustrate some of these advantages, Fig. 2 shows the tool position recorded during the drilling of a bovine femoral shaft when a constant thrust force of 25 N was set and the drill bit was rotating at 3000 rpm. This position signal is nearly free of noise compared to the force signal (Fig. 1). Very little or even no filtering is necessary. Drilling the cortical bone results in a slow feed rate, and the two layer transitions (from the first cortical wall to the cancellous tissue, and from the second cortical wall to outside the bone) are detected as an abrupt increase of the feed rate.

The thickness of the cortical walls can be estimated by the position of the tool (the ordinate in Fig. 2), approximately 7 mm in the two cases, while the cancellous tissue has a thickness of 24 mm. These distances can also be estimated using the impedance approach (reading the abscissa in Fig. 1 and know-
The admittance approach can be implemented by using an open-loop actuation, for example by applying a constant thrust force to the drilling tool (Fig. 4). From now on, in the Laplace domain, $G(s)$ represents the dynamics of the tool, whose output position is $X(s)$ after applying a force $F(s)$. This strategy was used with DRIBON to obtain the position response presented in Fig. 2.

![Figure 4: Open-loop admittance approach.](image)

The admittance approach can also be implemented in a different way by using a closed-loop algorithm consisting of a proportional controller with saturation (Fig. 5).

![Figure 5: Closed-loop admittance algorithm.](image)

The input reference is a ramp signal with slope $v_r$ imposing a constant feed rate on the tool. Thus, in the Laplace domain:

$$R(s) = \frac{v_r}{s^2}.$$  

(1)

The rest of the variables involved in the control algorithm are the position error $E(s)$ and the thrust force $F(s)$. Gain $K$ represents the P-controller. The output of this controller is saturated at a certain $f_{\text{max}}$ level before commanding the force. The block diagram also considers the existence of an external disturbance $F_c(s)$ which models the friction of the linear guide. Using Coulomb’s law, the kinetic friction is constant, independent of the velocity of motion:

$$F_c(s) = \frac{f_c}{s}.$$  

(2)

The effective thrust force applied to the bone is diminished by the friction. An experimental estimation of friction $f_c$ in the DRIBON system is presented in Appendix I.

Although the input of the control loop is a ramp-shaped position signal, it is important to note that the aim of the strategy is not to impose a constant feed rate within the bone, but to increase the thrust force gradually up to a constant value. Beyond this instant, the system behaves as if in an open loop (although the error is still computed).

To describe the operation of the admittance algorithm, two different situations arise depending on the saturation of the force. These two situations are described in the following subsections, together with the layer transition detection algorithm.

### 4.1. No force saturation

Fig. 6 shows a drilling experiment using a fresh bovine bone, where the feed rate is $v_r = 1$ mm/s, the proportional gain

![Figure 3: Third generation DRIBON tool developed by CEIT.](image)
$K = 11.25 \text{ N/mm}$ and force saturation $f_{\text{max}} = 23 \text{ N}$. The drill bit rotates at 3000 rpm and force saturation is not reached.

During the first eight seconds, the algorithm makes the tool approach the bone slowly ($v_r$ is this feed rate). The small error in this stage is mainly caused by friction (see Appendix I). When the drill bit comes into contact with the bone ($t \approx 8 \text{ s}$), the system has to overcome not only the friction in free movement but also the resistance of the bone itself. To increase the force the position error also has to increase. This increment is not arbitrarily large. Since the resistance force along the cortical wall oscillates within a bounded level (e.g., see Fig. 1), the error tends to a constant value with small fluctuations ($\varepsilon_{ss} \approx 1.6 \text{ mm}$).

From the control point of view, the proportional controller only adapts the resultant finite error to get the required thrust force for the imposed feed rate. Once the error and therefore the thrust force reach the required value, the system behaves as if in an open loop.

### 4.2. Force is saturated

Force saturation may occur, decreasing both the $f_{\text{max}}$ value and the rotational speed of the drill bit. Fig. 7 shows a new drilling experiment using the same control parameters as in the previous subsection, but with the drill bit rotating at 1000 rpm. With such speed and applying $f_{\text{max}}$ (this occurs for $t > 6 \text{ s}$), the resultant feed rate ($0.14 \text{ mm/s approx.}$) is much smaller than $v_r$.

It can be noted that as soon as the force is saturated, the closed-loop admittance algorithm behaves like the open-loop strategy. In this case, the position error increases linearly. From the control point of view, this unbounded increment of the error is not a deficiency of the controller. In this situation the only purpose of the controller is to gradually increase the force up to the saturation level. The error is computed because it is necessary for the layer transition detection.

### 4.3. Layer transition detection

In the previous experiments with and without saturation (Fig. 6 and Fig. 7), DRIBON stops at the end of the first cortical wall. We have seen that the parameters of the admittance algorithm affect the moment in which the system reaches a constant force and the final operating conditions in terms of force level and penetration rate. These conditions change when the system moves from the cortical wall to the cancellous tissue. In this transition the tendency of the tool is to rapidly increase its feed rate (as shown in Fig. 2).

Fig. 8 shows the derivative of the position error of the previous two experiments. The tendency of the tool to increase its feed rate can be detected as a strong decrease in this signal, thereby confirming the statements reported after examining Fig. 6 and Fig. 7. For example, during the last eight seconds in Fig. 8 (top), the signal oscillates around zero, meaning the position error is nearly constant. The signal at the bottom of Fig. 8 is nearly constant too, but at around $0.86 \text{ mm/s}$. That is why the feed rate of the tool is approximately $0.14 \text{ mm/s}$.

Regarding the threshold for stopping the drilling tool, because the signal has low noise, there is room to select different values. Note that if the threshold is set too close to zero, there are two moments in which the system could have stopped in the wrong place (at $t \approx 10 \text{ s}$ in the top graph in Fig. 8 and $t \approx 0.7 \text{ s}$ in the bottom graph in Fig. 8). By setting the threshold at around $-3 \text{ mm/s}$ our system works successfully, with no incorrect detections. However, there is room for further research on the experimental fluctuation of this signal in the material being drilled. It is also necessary to explore whether this signal should be filtered to eliminate or reduce these potentially conflicting situations.

Note that the efficacy of the layer transition detection does not depend on the selected parameters ($v_r$, $K$ and $f_{\text{max}}$) of the admittance algorithm. The tendency of the drilling tool to accelerate is clearly observed in the derivative of the position error.
Figure 8: Derivative of the position error in the case of no saturation (top) and in the case of saturation (bottom).

5. Experimental Performance

Having described the implementation and behavior of the closed-loop admittance algorithm, this section analyzes the experimental performance of the algorithm by focusing on two aspects: showing how far from or close to the layer transition the tool actually stops, and providing evidence that the algorithm works with different kinds of bones without collecting prior experimental data or retuning the parameters and threshold values. To show these two aspects experimentally, the drilling process ignores the first dip below the threshold and instead DRIBON stops at the end of the second cortical wall.

Fig. 9 shows the position of the tool using the following control parameters: ramp slope $v_r = 1$ mm/s, proportional gain $K = 11.25$ N/mm and force saturation $f_{\text{max}} = 40$ N. The drill bit has a diameter of 3.2 mm and rotates at 3000 rpm. With these values, the thrust force does not saturate.

It is interesting to note another benefit of the proposed closed-loop admittance algorithm with respect the open-loop implementation: in the latter case, the tool passes through the entire area of the internal soft tissue very fast (around time $t = 13$ s in Fig. 2). Using the ramp input, the tool quickly penetrates only the first 3 mm of the cancellous tissue. The last 14 mm of this part of the bone is drilled at the $v_r$ feed rate (from $t = 13$ s until $t = 27$ s).

Fig. 10 shows another experiment using the same parameters except force saturation $f_{\text{max}} = 30.6$ N. Now the thrust force does saturate. The system’s behavior within the cortical walls is slightly different because the error position increases over time, but the layer detection works as well as in the previous case. The fast jump within the soft tissue is a bit larger, and a small overshoot can be seen.

In both cases, with and without saturation, DRIBON is able to stop before the breakthrough of the bone. Fig. 11 shows the bovine femur (front and back view) after several drilling experiments. In all of them the drill bit stops before completing the breakthrough (the experiment shown in Fig. 9 produced the hole marked with the number 1, while the experiment presented
in Fig. 10 produced hole number 2). We can conclude that the
tendency of the tool to accelerate with the layer transition starts
before the drill bit crosses the boundary, and the automated sys-
tem is able to detect and react prior to breakthrough.

Figure 11: Front and back view of the bovine bone after several experiments.

With the position records, shown in Fig. 9 and Fig. 10, it
is possible to know the thickness of the bone and the cortical
walls, but it is not possible to know the thickness of the residual
layer of bone before the breakthrough. To show this qualita-
tively, Fig. 12 presents a cross section of the bone. Even in
this case it is difficult to measure thickness \( d \) (using a caliper,
\( d \approx 1.3 \) mm).

Figure 12: Cross-section of the bovine bone.

To show the robustness of the admittance algorithm, whose
performance is not sensitive to the characteristics of the bone,
the following set of experiments uses a fresh chicken bone.
Fig. 13 shows the penetration of the drilling tool inside this
bone. The diameter of the chicken bone is around 8 mm and
the cortical wall thickness is less than 2 mm. Once again, the
tool stops properly after the second cortical wall.

The control parameters were the same as in the previous ex-
periments with bovine femur (\( v_r = 1 \) mm/s, \( K = 11.25 \) N/mm
and \( f_{\text{max}} = 30.6 \) N). With these parameters and the bovine bone,
thrust force reached the saturation level. Now, with the chicken
bone, force is not saturated but it is not possible to see the sys-
tem reaching a constant level of error (and force) as shown in
Fig. 9. It seems that the cortical wall is thinner than the error
that would arise.

Fig. 14 shows the chicken bone after one experiment. A
small protrusion can be seen, but again the system is able to
stop before breakthrough. This experiment was repeated five
times, obtaining the same result each time. The size and prop-
erties of the chicken bone are different from the bovine bone,
but DRIBON works correctly without needing to tune the con-
trol parameters or change the threshold.

6. Conclusions

Automatic bone drilling is an active research field and has the
aim of reducing or even removing any undesired displacement
of the drill bit along the planned path. The goal is to provide
the surgeon with a smart mechatronic device able to drill the
desired depth and automatically stop before bone layer tran-
sitions occur or surrounding tissue are harmed. Most devices
and methodologies to date rely on force and torque sensors to
measure the penetration force and torque, and detect bone layer
transitions from these data. This strategy has many limitations:
1) measures are usually very noisy which complicates the con-
trol actuation, 2) multiple previous bone drills are necessary to
build proper neural networks and threshold values, 3) chang-
ing bone type requires that the strategy be readjusted and a new
neural network be built, and 4) it is very complicated to stop the
drill bit just before breakthrough occurs.

The current work describes in detail a very di-
fferent ap-
proach where only the drill displacement is measured. A
position sensor is only needed for bone layer detection and
the methodology is valid even if bone characteristics change,
which is largely more inexpensive than force and torque sen-
sors. Moreover, drilling can be stopped safely before break-
through. This methodology has been thoroughly described
throughout the paper. First of all, related work was reviewed and the differences between previous force-based methodologies and current position-based approaches were explained by using the impedance-admittance control analogy. Afterwards, the so-called admittance approach followed in this work was described in detail, showing the implications of each part of the algorithm for the drilling process.

Finally, the system and the proposed control algorithm were tested by drilling two very different types of bone, namely cow and chicken bone. Results of the experiments show how the system was able to use the same algorithm to stop the drilling process just before breaking through the second bone layer; no prior calibration or drilling experience was needed to ensure adequate performance.

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Appendix I

Prior to the drilling process, the tool is not in contact with the bone. The ramp input and the controller force the system to approach the bone with constant velocity \( v_r \). During this approaching phase, dynamics \( G(s) \) consists of the inertia of the moving part of drilling tool \( m \) and a small viscous damping \( b \) associated with the displacement along the linear guide:

\[
G(s) = \frac{1}{ms^2 + bs}
\]  

(3)

It is important to note that this is a type-I transfer function and therefore, given ramp input reference (1) and constant disturbance (2), the proportional controller produces a finite but small tracking error:

\[
e_{ss} = \lim_{s \to 0} s[R(s) - X(s)] = \frac{v_r b + f_c}{K} \approx \frac{f_c}{K}
\]  

(4)

Both the slope of the ramp \( v_r \) and the viscous damping \( b \) are so small that the error can be used to estimate the kinetic friction of the linear guide: \( f_c \approx e_{ss}K \).

Fig. 15 shows the experimental response of DRIBON to the closed-loop admittance algorithm. The tool follows the reference with a finite error of \( e_{ss} \approx 0.45 \text{ mm} \). Since the proportional gain is \( K = 11.25 \text{ N/mm} \), the friction can be estimated to be \( f_c \approx 5 \text{ N} \). This relatively high value is good for the system, because it allows DRIBON to point the drill bit in different directions with respect the horizontal (±20° approx.) without moving due to gravity force.

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